22.45. Model: A diffraction grating produces an interference pattern that is determined by both the slit spacing and the wavelength used.

Solve: An 800 line/mm diffraction grating has a slit spacing $d = (1.0 \times 10^{-3}\ \text{m})/800 = 1.25 \times 10^{-6}\ \text{m}$. Referring to Figure P22.45, the angle of diffraction is given by

$$\tan\theta_1 = \frac{y_1}{L} = \frac{0.436\ \text{m}}{1.0\ \text{m}} = 0.436 \Rightarrow \theta_1 = 23.557° \Rightarrow \sin\theta_1 = 0.400$$

Using the constructive-interference condition $d\sin\theta_m = m\lambda$,

$$\lambda = \frac{d\sin\theta_1}{1} = (1.25 \times 10^{-6}\ \text{m})(0.400) = 500\ \text{nm}$$

We can obtain the same value of λ by using the second-order interference fringe. We first obtain θ_2:

$$\tan\theta_2 = \frac{y_2}{L} = \frac{0.436\ \text{m} + 0.897\ \text{m}}{1.0\ \text{m}} = 1.333 \Rightarrow \theta_2 = 53.12° \Rightarrow \sin\theta_2 = 0.800$$

Using the constructive-interference condition,

$$\lambda = \frac{d\sin\theta_2}{2} = \frac{(1.25 \times 10^{-6}\ \text{m})(0.800)}{2} = 500\ \text{nm}$$

Assess: Calculations with the first-order and second-order fringes of the interference pattern give the same value for the wavelength.